Universität Tübingen
Fachbereich Mathematik

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07.11.24

Masstheoretische Methoden
WS 2024/25
3.Übung

AUFGABE 6:
Let $X := \sum_{i \in I} \mathbb{R} = \{(x, i) \mid x \in \mathbb{R}, i \in I\}$ be the topological sum of real axes with respect to an index set I, in particular X is a metric space. For $A \subseteq X, i \in I$, we write $A_i := A \cap (\mathbb{R} \times \{i\})$ and define

$$\mu(A) := \sum_{i \in I} \mathcal{L}^1(A_i).$$

Verfiy that μ is a borelregular measure on X. Show for uncountable I that there is a closed set $A \subseteq X$ with

$$\mu(A) < \inf_{U \supseteq A \text{ offen}} \mu(U).$$

AUFGABE 7: (Vitali's covering theorem, finite version)
Let $\mathcal{F}$ be a finite family of closed, non-degenerate balls in a metric space X. Show that there exists a pairwise disjoint subfamily $\mathcal{G} \subseteq \mathcal{F}$ with

$$\cup_{B \in \mathcal{F}} B \subseteq \cup_{B' \in \mathcal{G}} \tilde{B}',$$

where $\tilde{B}'$ denotes a closed ball with three times the radius and same center as B'.

Abgabetermin ist Donnerstag, 14.11.24.