Talks on Motives

The category $\mathcal{M}$ of motives is a hypothetical abelian category through which all classical cohomology theories for varieties should factor by an additive functor. In that way, assertions like the Künneth decomposition (and much more) should hold in $\mathcal{M}$ thus yielding Künneth formulas for all cohomology theories at once.

1. **Introduction to motives** **Paul Vögele**
   de Rham and singular cohomology and comparison, Grothendieck sites and sheaf cohomology, de Rham in positive characteristic, Witt vectors, lift and crystalline cohomology, Chow ring, correspondences, desired properties of motives, effective motives

   [11] and [3, 12, 20]

2. **Tannakian categories** **Giacomo Gavelli**
   Tannaka-Krein duality and locally constant sheaves, monoidal categories, tensor categories, main theorem

   [4] and [16, 17]

3. **The standard conjectures** **Clemens Nollau**
   the Weil conjectures, Weil cohomology, correspondences, Lefschetz and Hodge conjectures, permanence properties

   [8], for the last point: [1], general: [21]

4. **Classical motives** **Dan Funck**
   rational equivalence, Chow groups, pseudo abelian categories, rational motives, Manin's identity principle, curves and abelian varieties

   [18] and [7, 9]

5. **Voevodsky's derived category of motives** **Felix Röhrle**
   [19], see also [2]

6. **Motivic cohomology and Beilinson's conjecture** **Anton Deitmar**
   [15], Chapter 2.

More material in [10, 13, 14]

References

[1] D. Arapura, *Motivation for Hodge cycles*, Advances in Mathematics **207** (2006), available at `arXiv:math/0501348`.

[2] Alexander Beilinson and Vadim Vologodsky, *A DG guide to Voevodsky's motives* (2008), available at `http://pages.uoregon.edu/vvologod/papers/motvo.pdf`.

[3] P. Berthelot, *Cohomologie cristalline des schémas de caractéristique p¿0* (1974).

[4] P. Deligne and J. Milne, *Tannakian categories*, available at `https://www.jmilne.org/math/xnotes/tc.pdf`.

https://www.jmilne.org/math/xnotes/tc.pdf

[5] Masaki Kashiwara and Pierre Schapira, *Sheaves on manifolds*, Grundlehren der Mathematischen Wissenschaften [Fundamental Principles of Mathematical Sciences], vol. 292, Springer-Verlag, Berlin, 1994. With a chapter in French by Christian Houzel; Corrected reprint of the 1990 original. MR1299726 (95g:58222)

[6] ———, *Categories and sheaves*, Grundlehren der Mathematischen Wissenschaften [Fundamental Principles of Mathematical Sciences], vol. 332, Springer-Verlag, Berlin, 2006. MR2182076 (2006k:18001)

[7] Steven L. Kleiman, *Motives*, Algebraic geometry, Oslo 1970 (Proc. Fifth Nordic Summer School in Math.), Wolters-Noordhoff Publishing, Groningen, 1972, pp. 53–82.

[8] ———, *The standard conjectures*, Motives (Seattle, WA, 1991), Proc. Sympos. Pure Math., vol. 55, Amer. Math. Soc., Providence, RI, 1994, pp. 3–20.

[9] Ju. I. Manin, *Correspondences, motifs and monoidal transformations*, Mat. Sb. (N.S.) **77(119)** (1968), 475–507 (Russian).

[10] Carlo Mazza, Vladimir Voevodsky, and Charles Weibel, *Lecture notes on motivic cohomology*, Clay Mathematics Monographs, vol. 2, American Mathematical Society, Providence, RI, 2006. MR2242284 (2007e:14035)

[11] James Milne, *Motives - Grothendieck's dream* (2012), available at `http://www.jmilne.org/math/xnotes/MOT.pdf`.

[12] ———, *Lectures on etale cohomology*, available at `https://www.jmilne.org/math/CourseNotes/lec.html`.

https://www.jmilne.org/math/CourseNotes/lec.html

[13] Uwe Jannsen, Steven Kleiman, and Jean-Pierre Serre (eds.), *Motives*, Proceedings of Symposia in Pure Mathematics, vol. 55, American Mathematical Society, Providence, RI, 1994. MR1265518 (94i:11003)

[14] ——— (ed.), *Motives*, Proceedings of Symposia in Pure Mathematics, vol. 55, American Mathematical Society, Providence, RI, 1994. MR1265549 (94i:11004)

[15] Kartik Prasanna and Akshay Venkatesh, *Automorphic cohomology, motivic cohomology, and the adjoint L-function*, Astérisque **428** (2021), viii+132, DOI 10.24033/ast (English, with English and French summaries). MR4372499

[16] Neantro Saavedra Rivano, *Catégories Tannakiennes*, Lecture Notes in Mathematics, Vol. 265, Springer-Verlag, Berlin, 1972 (French).

[17] ———, *Catégories tannakiennes*, Bull. Soc. Math. France **100** (1972), 417–430 (French).

[18] A. J. Scholl, *Classical motives*, Motives (Seattle, WA, 1991), Proc. Sympos. Pure Math., vol. 55, Amer. Math. Soc., Providence, RI, 1994, pp. 163–187. MR1265529 (95b:11060)

[19] Vladimir Voevodsky, Andrei Suslin, and Eric M. Friedlander, *Cycles, transfers, and motivic homology theories*, Annals of Mathematics Studies, vol. 143, Princeton University Press, Princeton, NJ, 2000. MR1764197 (2001d:14026)

[20] *Wikipedia on motives*, available at `https://en.wikipedia.org/wiki/Motive_(algebraic_geometry)`.

[21] *Wikipedia on the Standard Conjectures*, available at `https://en.wikipedia.org/wiki/Standard_conjectures_on_algebraic_cycles`.