Propagation of Chaos

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Sheet 12

Exercise 1: Show that there is a constant $C < \infty$ such that for any $N \in \mathbb{N}$, any $\Psi \in L^2\left(\mathbb{R}^{3N}\right)$ and any $\phi \in \mathbb{R}^3$

$$\left\|\mu^{\Psi} - p^{\phi}\right\|_{tr} \leq \left\|q^{\phi}\Psi\right\| \left(\left\|q^{\phi}\Psi\right\| + \left\|\widehat{\mathbb{1}^{\phi}_{odd}}\Psi\right\|\right)$$

Exercise 2: Let $\Psi_t \in L^2(\mathbb{R}^{3(N+M)})$ be a solution of the Schrödinger equation

$$\begin{aligned} i\frac{d}{dt}\Psi_t(x_1,\ldots,x_N,y_1,\ldots,y_M) = &-\sum_{j=1}^{N}\Delta_{x_j} - \sum_{j=1}^{M}\Delta_{y_j} + \frac{1}{N-1}\sum_{j<k;j,k=1}^{N} V(x_j - x_k) \\ &+ \frac{1}{M-1}\sum_{j<k;j,k=1}^{M} W(y_j - y_k) + \frac{1}{N+M}\sum_{j=1}^{N}\sum_{k=1}^{M} U(x_j - y_k) \end{aligned}$$

with initial state $\Psi_0 := \prod_{j=1}^{N}\phi_0(x_j)\prod_{j=1}^{M}\eta_0(y_j)$ for some one-body states ϕ_0 and η_0. Derive on a heuristic level the respective effective description of the system for $M, N \to \infty$.

Exercise 3: Show that for any $t > 0$ the expression $\langle \Psi_t, \left(q^{\phi_t}_{x_1} + q^{\eta_t}_{y_1}\right)\Psi_t\rangle$ tends to zero as N and M go to infinity based on the setting in exercise 2.