Propagation of Chaos

Prof. Dr. P. Pickl

Sheet 7

Aufgabe 1: (Relative Entropy)

(a) Show that for continuous probability densities, the relative entropy stays invariant under parameter transformations.

(b) Show that the relative entropy is convex in the first argument, i.e. for an $\lambda \in [0,1]$

$$r(\lambda\rho_1 + (1-\lambda)\rho_2, \eta) \leq \lambda r(\rho_1, \eta) + (1-\lambda) r(\rho_2, \eta) \;.$$

Aufgabe 2: (Maxweel Boltzmann distribution) Consider a system of k boxes containing in total N particles. A macro-state for the system is defined via the numbers $n_1, n_2 \ldots, n_k$ of particles in each box. Assume that the energy of each particle is determined by the box where the particle is found, i.e. the total energy is given by $\sum_{j=1}^{k} n_j e_j$ for given numbers $e_1 \leq e_2 \leq \ldots \leq e_k \in \mathbb{R}$

For which macro-state is the entropy maximal under the assumption that the total particle number N and the total energy of the system E with $e_1 \leq E \leq e_k$ are given.

Compare your result with the well known Maxwell–Boltzmann distribution.

Aufgabe 3: (Collision operator)

Calculate the probability density $W(\overline{v}, \overline{w}, \widetilde{v}, \widetilde{w})$ describing the probability that the outgoing momenta after tho collision are given by $\widetilde{v}$ and $\widetilde{w}$ under the condition that two particles have collided during an infinitesimal time-interval dt with ingoing velocities $\overline{v}$ and $\overline{w}$.