Propagation of Chaos

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Sheet 6

Aufgabe 1: Verify the formula

$$\frac{dS}{dU} = \frac{1}{T}$$

for an ideal gas of N particles in a box. Here S is the Boltzmann entropy, T the temperature and U the heat transferred to the gas.

Hint: Think of the macro-state of all configurations with a total kinetic energy of the gas particles less or equal to some E. You can assume that E is proportional to T. Change the macro state by changing the maximal kinetic energy to $E + dU$. Calculate the number of micro-states in both cases and the change in the entropy. Verify is $\frac{dS}{dU}$ proportional to $\frac{1}{E}$.

Aufgabe 2: Prove that the relative entropy is neither symmetric nor satisfies the triangle-inequality on the space of all continuous functions $f : [0, 1] \to \mathbb{R}_0^+$.

Aufgabe 3: Let $\mathcal{F}_t : \mathbb{R}^{6N} \to \mathbb{R}_0^+$ be the probability density of the Newtonian system discussed in class. In particular $\mathcal{F}_0(X, V) = \prod_{j=1}^N \rho_0(x_j, v_j)$ and $\mathcal{F}_t(X_t) = \mathcal{F}_0(X_0)$.

Let $\overline{\mathcal{F}}_t := \prod_{j=1}^N \rho_t(x_j, v_j)$ where ρ_t is a solution of the Vlasov equation. Assume that $\ln \rho_t$ is globally Lipschitz for all times.

Estimate the relative entropy between $\overline{\mathcal{F}}_t$ and $\mathcal{F}_t$ in terms of N, $\|X_t - \overline{X}_t\|_\infty$ and $\| \ln \rho_t\|_L$.