Propagation of Chaos

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Blatt 2

Aufgabe 1: (Brownian motion I)

Consider the model for Brownian motion we discussed in class. Show that the approximation for the empirical density of the system we got using the Moivre-Laplace Theorem (Central Limit Theorem) in fact solves the heat equation.

Aufgabe 2: (Brownian motion II)

Again, consider the model for Brownian motion of N particles on $\mathbb{Z}$ we discussed in class. Assume that at each time step each particle either stays at its position (with probability 1/2) makes one steep to the left (with probability 1/3) or makes one steep to the left (with probability 1/6). Again we assume independence among all particles and time-steps.

Formulate the PDE which approximates the empirical density and show that, taking first the limit $N \to \infty$ and then $t \to \infty$, the solution of the PDE in fact gives a point-wise approximation of the empirical density.

Aufgabe 3: Let $\Phi_t : \mathbb{R}^{dN} \to \mathbb{R}^{dN}$ be the Newtonian flow, i.e. $\Phi_t(X_0) = X_t$, where X_t is the solution of the Newtonian equations of motion: $\dot{x}_t^j = v_t^j$ and $\dot{v}_t^j = \sum_{k \neq j} f(x_j - x_k)$ with $f = \nabla V$ and initial condition $X_{t=0} = X_0$.

Show that for any $t \in \mathbb{R}$, any $p \in [1, \infty]$ and any function $f \in L^p$ it holds that

$$\|f\|_p = \|f \circ \Phi_t\|_p \ .$$