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Winter semester 2023/24
November 21, 2023

Foundations of Quantum Mechanics: Assignment 6

Exercise 22: Essay question. (20 points)
Explain the concept of contextuality for the example of a quantum measurement of σ_3.

Exercise 23: Spinors (30 points)
Verify that $|\boldsymbol{\omega}(\phi)| = \|\phi\|_S^2 = \phi^*\phi$. Proceed as follows: By (2.108), $\boldsymbol{\omega}(z\phi) = |z|^2\boldsymbol{\omega}(\phi)$, it suffices to show that unit spinors are associated with unit vectors. By (2.108) again, it suffices to consider ϕ with $\phi_1 \in \mathbb{R}$ (else replace ϕ by $e^{i\theta}\phi$ with appropriate θ). So we can assume, without loss of generality, $\phi = (\cos\alpha, e^{i\beta}\sin\alpha)$ with $\alpha, \beta \in \mathbb{R}$. Evaluate $\phi^*\boldsymbol{\sigma}\phi$ explicitly in terms of α and β, using the explicit formulas (2.105) for $\boldsymbol{\sigma}$. Then check that it is a unit vector.

Exercise 24: Half Angles (30 points)
(a) Show that for unit vectors ϕ, χ in spin space S,

$$2\big|\langle\phi|\chi\rangle\big|^2 = 1 + \sum_{a=1}^{3}\langle\phi|\sigma_a\phi\rangle\langle\chi|\sigma_a\chi\rangle\,.$$

(b) Conclude further that if ϕ and χ have angle $\theta = \arccos\big|\langle\phi|\chi\rangle\big|$ in S, then $\boldsymbol{\omega}(\phi)$ and $\boldsymbol{\omega}(\chi)$ have angle 2θ in $\mathbb{R}^3$.

Exercise 25: Can't Distinguish Non-Orthogonal State Vectors (20 points)
(a) Alice gives to Bob a single particle whose spin state ψ is either $(1,0)$ or $(0,1)$ or $\frac{1}{\sqrt{2}}(1,1)$. Bob can carry out a quantum measurement of an arbitrary self-adjoint operator. Show that it is impossible for Bob to decide with certainty which of the three states ψ is.

(b) The same with only $(1,0)$ and $\frac{1}{\sqrt{2}}(1,1)$.

Hand in: By Tuesday November 28, 2023, 8:15 am

Reading assignment due Thursday November 30, 2023: T. Maudlin, Three Measurement Problems. *Topoi* **14(1)**: 7–15 (1995). Read pages 7–12 and the first two paragraphs on page 13.