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Winter semester 2023/24
November 7, 2023

Foundations of Quantum Mechanics: Assignment 4

Exercise 14: Essay question. (25 points)
Describe the delayed-choice double-slit experiment. Why does it seem paradoxical? How does the paradox get resolved in Bohmian mechanics?

Exercise 15: Galilean relativity of Bohmian mechanics (25 points)
A Galilean change of space-time coordinates ("Galilean boost") is given by

$$\boldsymbol{x}' = \boldsymbol{x} + \boldsymbol{v}t\,, \quad t' = t \tag{1}$$

with a constant $\boldsymbol{v} \in \mathbb{R}^3$ called the relative velocity. Suppose the potential V is translation invariant. In some previous problem it was shown that if $\psi(t, \boldsymbol{x}_1, \ldots, \boldsymbol{x}_N)$ is a solution of the Schrödinger equation, then so is

$$\psi'(t', \boldsymbol{x}'_1, \ldots, \boldsymbol{x}'_N) = \exp\left[\tfrac{i}{\hbar}\sum_{i=1}^{N} m_i(\boldsymbol{x}'_i \cdot \boldsymbol{v} - \tfrac{1}{2}\boldsymbol{v}^2 t')\right] \psi\Big(t', \boldsymbol{x}'_1 - \boldsymbol{v}t', \ldots, \boldsymbol{x}'_N - \boldsymbol{v}t'\Big). \tag{2}$$

Now show that if $t \mapsto (\boldsymbol{Q}_1, \ldots, \boldsymbol{Q}_N)$ is a solution of Bohmian mechanics then so is $t \mapsto (\boldsymbol{Q}'_1, \ldots, \boldsymbol{Q}'_N)$.

Exercise 16: Conserved operator (30 points)
Show that if the potential $V(\boldsymbol{x}_1, \ldots, \boldsymbol{x}_N)$ is C^1 and translation invariant [as in (1.64) with $R = I$], then each component of the total momentum operator

$$P_a = -i\hbar \sum_{j=1}^{N} \frac{\partial}{\partial x_{ja}} \tag{3}$$

$(a = 1, 2, 3)$ commutes with the Hamiltonian

$$H = -\sum_{j=1}^{N} \frac{\hbar^2}{2m_j} \nabla_j^2 + V \tag{4}$$

when acting on $\psi \in C^3 \cap L^2$. Show further that if V is C^1 and rotation invariant [as in (1.64) with $\boldsymbol{a} = \boldsymbol{0}$], then each component of the total angular momentum operator

$$\boldsymbol{L} = -i\hbar \sum_{j=1}^{N} \boldsymbol{x}_j \times \nabla_j \tag{5}$$

commutes with H when acting on $\psi \in C^3 \cap L^2$.

Exercise 17: Fourier transform
(a) (20 points) Find the Fourier transform $\widehat{\psi}$ of the function

$$\psi(x) = \begin{cases} 0 & x < -1 \\ 2^{-1/2} & -1 \le x \le 1 \\ 0 & x > 1, \end{cases} \tag{6}$$

where x is a 1-dimensional variable.

(b) (optional, 15 extra points) Plot $\widehat{\psi}$ using a computer.

(c) (optional, 15 extra points) Using suitable software (such as Mathematica, Maple, or Matlab), make the computer find the formula for $\widehat{\psi}$. (That is, you need to find the command for symbolically computing Fourier transforms and the one for defining a function like (6), and run them.)

Hand in: by Tuesday, November 14, 2023, 8:15am

No reading assignment this week.