22

Are there quantum jumps?

If we have to go on with these damned quantum jumps, then I'm sorry that I ever got involved. *E. Schrödinger*

1 Introduction

I have borrowed the title of a characteristic paper by Schrödinger (Schrödinger, 1952). In it he contrasts the smooth evolution of the Schrödinger wavefunction with the erratic behaviour of the picture by which the wavefunction is usually supplemented, or 'interpreted', in the minds of most physicists. He objects in particular to the notion of 'stationary states', and above all to 'quantum jumping' between those states. He regards these concepts as hangovers from the old Bohr quantum theory, of 1913, and entirely unmotivated by anything in the mathematics of the new theory of 1926. He would like to regard the wavefunction itself as the complete picture, and completely determined by the Schrödinger equation, and so evolving smoothly without 'quantum jumps'. Nor would he have 'particles' in the picture. At an early stage, he had tried to replace 'particles' by wavepackets (Schrödinger, 1926). But wavepackets diffuse. And the paper of 1952 ends, rather lamely, with the admission that Schrödinger does not see how, for the present, to account for particle tracks in track chambers... nor, more generally, for the definiteness, the particularity, of the world of experience, as compared with the indefiniteness, the waviness, of the wavefunction. It is the problem that he had had (Schrödinger, 1935*a*) with his cat. He thought that she could not be both dead and alive. But the wavefunction showed no such commitment, superposing the possibilities. Either the wavefunction, as given by the Schrödinger equation, is not everything, or it is not right.

Of these two possibilities, that the wavefunction is not everything, or not right, the first is developed especially in the de Broglie–Bohm 'pilot wave' picture. Absurdly, such theories are known as 'hidden variable' theories. Absurdly, for there it is not in the wavefunction that one finds an image of the visible world, and the results of experiments, but in the complementary 'hidden'(!) variables. Of course the extra variables are not confined to the visible 'macroscopic' scale. For no sharp definition of such a scale could be made. The 'microscopic' aspect of the complementary variables is indeed hidden from us. But to admit things not visible to the gross creatures that we are is, in my opinion, to show a decent humility, and not just a lamentable addiction to metaphysics. In any case, the most hidden of all variables, in the pilot wave picture, is the wavefunction, which manifests itself to us only by its influence on the complementary variables.

If, with Schrödinger, we reject extra variables, then we must allow that his equation is not always right. I do not know that he contemplated this conclusion, but it seems to me inescapable. Anyway it is the line that I will follow here. The idea of a small change in the mathematics of the wavefunction, one that would little affect small systems, but would become important in large systems, like cats and other scientific instruments, has often been entertained. It seems to me that a recent idea (Ghirardi, Rimini and Weber, 1985), a specific form of spontaneous wavefunction collapse, is particularly simple and effective. I will present it below. Then I will consider what light it throws on another of Schrödinger's preoccupations. He was one of those who reacted most vigorously (Schrödinger, 1935*a*, *b*, 1936) to the famous paper of Einstein, Podolsky and Rosen (1935). As regards what he called 'quantum entanglement', and the resulting EPR correlations, he 'would not call that *one* but rather *the* characteristic trait of quantum mechanics, the one that enforces its entire departure from classical lines of thought'.

2 Ghirardi, Rimini and Weber

The proposal of Ghirardi, Rimini and Weber, is formulated for non-relativistic Schrödinger quantum mechanics. The idea is that while a wavefunction

$$\psi(t, \mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_N) \tag{1}$$

normally evolves according to the Schrödinger equation, from time to time it makes a jump. Yes, a jump! But we will see that these GRW jumps have little to do with those to which Schrödinger objected so strongly. The only resemblance is that they are random and spontaneous. The probability per unit time for a GRW jump is

$$\frac{N}{\tau}, \tag{2}$$

where N is the number of arguments $\mathbf{r}$ in the wavefunction, and τ is a new constant of nature. The jump is to a 'reduced' or 'collapsed' wavefunction

$$\psi' = \frac{j(\mathbf{x} - \mathbf{r}_n)\psi(t, \ldots)}{R_n(\mathbf{x})}, \tag{3}$$

where $\mathbf{r}_n$ is randomly chosen from the arguments $\mathbf{r}$. The jump factor j is normalized:

$$\int \mathrm{d}^3\mathbf{x}\,|j(\mathbf{x})|^2 = 1. \tag{4}$$

Ghirardi, Rimini and Weber suggest a Gaussian:

$$j(\mathbf{x}) = K\exp(-\mathbf{x}^2/2a^2) \tag{5}$$

where a is again a new constant of nature. R is a renormalization factor:

$$|R_n(\mathbf{x})|^2 = \int \mathrm{d}^3\mathbf{r}_1 \cdots \mathrm{d}^3\mathbf{r}_N\,|j\psi|^2. \tag{6}$$

Finally the collapse centre $\mathbf{x}$ is randomly chosen with probability distribution

$$\mathrm{d}^3\mathbf{x}\,|R_n(\mathbf{x})|^2. \tag{7}$$

For the new constants of nature, GRW suggest as orders of magnitude

$$\tau \approx 10^{15}\,\mathrm{s} \approx 10^8\,\mathrm{year} \tag{8}$$

$$a \approx 10^{-5}\,\mathrm{cm}. \tag{9}$$

An immediate objection to the GRW spontaneous wavefunction collapse is that it does not respect the symmetry or antisymmetry required for 'identical particles'. But this will be taken care of when the idea is developed in the field theory context, with the GRW reduction applied to 'field variables' rather than 'particle positions'. I do not see why that should not be possible, although novel renormalization problems may arise.

There is no problem in dealing with 'spin'. The wavefunctions ψ and ψ' in (3) can be supposed to carry suppressed spin indices.

Consider now the wavefunction

$$\phi(\mathbf{s}_1 \cdots \mathbf{s}_L)\chi(\mathbf{r}_1 \cdots \mathbf{r}_M), \tag{10}$$

where L is not very big and M is very very big. The first factor, ϕ, might represent a small system, for example an atom or molecule, that is temporarily isolated from the rest of the world... the latter, or part of it, represented by the second factor, χ. The GRW process for the complete wavefunction implies independent GRW processes for the two factors. From (8) we can forget about GRW processes in the small system. But in the big system, with M of order say 10^{20} or larger, the mean lifetime before a

GRW jump is some

$$\frac{10^{15}}{10^{20}} = 10^{-5}\,\text{s} \tag{11}$$

or less.

Consider next a wavefunction like

$$\phi_1(\mathbf{s}_1 \cdots \mathbf{s}_L)\chi_1(\mathbf{r}_1 \cdots \mathbf{r}_M) + \phi_2(\mathbf{s}_1 \cdots \mathbf{s}_L)\chi_2(\mathbf{r}_1 \cdots \mathbf{r}_M). \tag{12}$$

This might represent the aftermath of a 'quantum measurement' situation. Some 'property' of the small system has been 'measured' by interaction with a large 'instrument', which is thrown as a result into one or other of the states χ_1 or χ_2, corresponding to different pointer readings. This macroscopic difference between χ_1 and χ_2 implies that, for very many arguments $\mathbf{r}$, multiplication of the wavefunction by $j(\mathbf{x} - \mathbf{r})$ will reduce to zero one or other of the terms in (12). Thus in a time of order (11) one of the terms will disappear, and only the other will propagate. The wavefunction commits itself very quickly to one pointer reading or the other. Moreover, the probability that one term rather than the other survives is proportional to the fraction of the total norm which it carries – in agreement with the rule of pragmatic quantum theory.

Quite generally any embarrassing macroscopic ambiguity in the usual theory is only momentary in the GRW theory. The cat is not both dead and alive for more than a split second. One could worry perhaps if the GRW process does not go too far. In the usual pragmatic theory the 'reduction' or 'collapse' of the wavefunction is an operation performed by the theorist at some time convenient for her. Usually she will delay this till the Schrödinger equation has established a very big difference between χ_1 and χ_2. The GRW process is one of nature, and comes about as soon as the difference between χ_1 and χ_2 is big enough. I think that with suitable values of the natural constants (8, 9) the GRW theory will nevertheless agree with the pragmatic theory in practice. But studies on models would be useful to build up confidence in this.

3 Quantum entanglement

There is nothing in this theory but the wavefunction. It is in the wavefunction that we must find an image of the physical world, and in particular of the arrangement of things in ordinary three-dimensional space. But the wavefunction as a whole lives in a much bigger space, of $3N$-dimensions. It makes no sense to ask for the amplitude or phase or whatever of the wavefunction at a point in ordinary space. It has neither amplitude nor phase nor anything else until a multitude of points in ordinary threespace are specified. However, the GRW jumps (which are part of the wavefunction, not something else) are well localized in ordinary space. Indeed each is centred on a particular specetime point $(\mathbf{x}, t)$. So we can propose these events as the basis of the 'local beables' of the theory. These are the mathematical counterparts in the theory to real events at definite places and times in the real world (as distinct from the many purely mathematical constructions that occur in the working out of physical theories, as distinct from things which may be real but not localized, and as distinct from the 'observables' of other formulations of quantum mechanics, for which we have no use here). A piece of matter then is a galaxy of such events. As a schematic psychophysical parallelism we can suppose that our personal experience is more or less directly of events in particular pieces of matter, our brains, which events are in turn correlated with events in our bodies as a whole, and they in turn with events in the outer world.

In this paper we will use the notion of localization of events only in a rough way. We will localize them in one or other of two widely separated regions of space which we suppose to be occupied by two widely separated systems.

Let the arguments $\mathbf{s}$ and $\mathbf{r}$ in (12) refer to the two sides, respectively, in an Einstein–Podolsky–Rosen–Bohm setup, with L as well as M now large. A source, which for simplicity we omit from the analysis, emits a pair of spin $-\frac{1}{2}$ neutrons in the singlet spin state. They move through Stern–Gerlach magnets to counters which register for each neutron whether it has been deflected 'up' or 'down' in the corresponding magnet. According to the Schrödinger equation the wavefunction would come out like (12), with ϕ_1 or ϕ_2 corresponding to 'up' or 'down' on the left, and χ_1 or χ_2 corresponding to 'down' or 'up' on the right. Suppose that the left hand counters are closer to the source, and so register before the right hand ones. That is to say, suppose that ϕ_1 differs macroscopically from ϕ_2 before χ_1 from χ_2. Then the GRW jumps on the left quickly reduce the wavefunction to one or other of the two terms in (12). The choice between χ_1 and χ_2, as well as between ϕ_1 and ϕ_2, has then been made. The jumps on the left are decisive, and those on the right have no opportunity to be so.

In all this the GRW account is very close to that of a common way of presenting conventional quantum mechanics, with 'measurement' causing 'wavefunction collapse' – and with a 'measurement' somewhere causing 'collapse' everywhere. But it is important that in the GRW theory everything, including 'measurement', goes according to the mathematical equations of the theory. Those equations are not disregarded from time to time on the basic of supplementary, imprecise, verbal, prescriptions.

In this EPRB situation, an 'up' on the left implies a subsequent 'down' on the right, and vice versa. Now of course it was not the existence of correlations between distant events that scandalized EPR, and led Einstein (Einstein, 1949) to use the word 'paradox' in this connection. Such correlations are common in daily life. If I find that I have brought only one glove, the left handed, then I confidently predict that the one at home will be found to be right handed. In the everyday conception of things there is no puzzle here. Both gloves have been there all morning, and each has been right or left handed all the time. Observation of the one taken from my pocket gives information about, but does not influence, the one left at home. As regards EPRB correlations, what is disturbing about quantum mechanics, especially as sharpened by GRW, is that before the first 'measurement' there *is* nothing but the quantum mechanical wavefunction – entirely neutral between the two possibilities. The decision between these possibilities is made for both of the mutually distant systems only by the first 'measurement' on one of them. There is no question, if there *was* nothing but the wavefunction, of just revealing a decision already taken. It was this 'spooky action at a distance', the immediate determining of events in a distant system by events in a near system, that scandalized EPR. They concluded that quantum mechanics must, at best, be incomplete. There must be in nature additional variables, not yet known to quantum mechanics, in both systems, which determine in advance the results of experiments, and which happen to have become correlated at the source – just as gloves happen to be sold in matching pairs.

It is now very difficult to maintain this hope, that local causality might be restored to quantum mechanics by the addition of complementary variables. The perfect correlations actually considered by EPR, with parallel polarizers in the EPRB setup, do not present any difficulty in this respect. But the imperfect correlations implied by quantum mechanics, for misaligned polarizers, prove more intractable (e.g. Bell, 1981).

The GRW theory does not add variables. But by adding mathematical precision to the jumps in the wavefuction, it seems simply to make precise the action at a distance of ordinary quantum mechanics. The most disturbing aspect of this is the apparent difficulty of reconciling it with Lorentz invariance. For in a Lorentz invariant theory we tend to think that 'nothing goes faster than light'. So we turn now to a discussion of Lorentz invariance.

4 Relative time translation invariance

Of course we cannot discuss full Lorentz invariance in the context of the nonrelativistic model presented above. But there is a residue, or at least an analogue, of Lorentz invariance, which can be discussed in the case of two widely separated systems. Consider the Lorentz transformation

$$z' = \gamma(z - vt), \quad t' = \gamma(t - vz) \tag{13}$$

with x and y unchanged, where the velocity of light has been set equal to unity, and

$$\gamma = \frac{1}{(1 - v^2)^{1/2}}. \tag{14}$$

In the case of a system at a large distance, a, from the origin, it is convenient to introduce a new origin, so that

$$z \rightarrow z + a. \tag{15}$$

Then (13) becomes

$$z' = -a + \gamma(z + a - vt), \quad t' = \gamma(t - v(z + a)). \tag{16}$$

Taking v very small and a very large so that

$$va = k \tag{17}$$

(16) becomes

$$z' = z, \quad t' = t - k. \tag{18}$$

In the case of a single system this tells us simply to expect invariance with respect to translation in time. But in the case of two systems displaced from the origin in opposite directions, and so with different signs for k, it tells us to expect invariance with respect to displacement in *relative* time.

Multiple time formalism, with independent times for different particles, or for different points in space, is an old story in relativistic quantum theory. It is less familiar in the context of the nonrelativistic theory. However, it is easily implemented *in the case of noninteracting systems* at the level of the Schrödinger equation. Let two noninteracting subsystems have separate Hamiltonians A and B, respectively, so that the total Hamiltonian is

$$H = A + B. \tag{19}$$

Then from the ordinary one-time wavefunction $\psi(t, \ldots)$ we can define a two-time wavefunction

$$\psi(t', t'', \ldots) = \frac{\exp \mathrm{i}(t - t')A}{\hbar} \frac{\exp \mathrm{i}(t - t'')B}{\hbar} \psi(t, \ldots). \tag{20}$$

Since A and B commute, the relative order of the two exponentials in (20) is unimportant. (However, if A and B are time-dependent, the two exponentials must separately be time ordered, as in (A.5)). The two-time wavefunction satisfies the two Schrödinger equations

$$\frac{\hbar i\partial}{\partial t'}\psi(t', t''\ldots) = A\psi(t', t'', \ldots) \tag{21}$$

$$\frac{\hbar i\partial}{\partial t''}\psi(t', t''\ldots) = B\psi(t', t'', \ldots). \tag{22}$$

These equations are invariant against independent shifts in the origins of the two time variables (provided any time dependent external fields in A and B are shifted appropriately).

It remains to see if this relative time invariance survives the introduction of the GRW jumps. It does. I did not find a short elegant argument, and have relegated the clumsy arguments that I did find to an appendix. From the ordinary one-time wavefunction for time i, a two-time wavefunction can again be constructed. It incorporates the jumps of subsystem-1 between times i and i', and those of subsystem-2 between i and i''. In terms of this a formula can be found (A.22, A.23) for the probability of subsequent jumps before times f' and f'' in the two subsystems respectively. It can be interpreted as supplementing (21, 22) by giving the probabilities for jumps in the two systems as t' and t'' are advanced independently from independent starting points. It does not depend on t' or t'' except through the two-time wavefunction ψ (and any time dependent external fields in Hamiltonians A and B). The relative time translation invariance of the theory is then manifest.

The reformulation (A.22, A.23) of the theory can also be used to calculate the statistics of jumps in one system separately, disregarding what happens in the other. The result (A.24, A.25) makes no reference to the second system. Events in one system, considered separately, allow no inference about events in the other, nor about external fields at work in the other,... nor even about the very existence of the other system. There are no 'messages' in one system from the other. The inexplicable correlations of quantum mechanics do not give rise to signalling between noninteracting systems. Of course, however, there may be correlations (e.g. those of EPRB) and if something about the second system is given (e.g. that it is the other side of an EPRB setup) and something about the overall state (e.g. that it is the EPRB singlet state) then inferences from events in one system (e.g. 'yes' from the 'up' counter) to events in the other (e.g. 'yes' from the 'down' counter) are possible.

5 Conclusion

I think that Schrödinger could hardly have found very compelling the GRW theory as expounded here – with the arbitrariness of the jump function, and the elusiveness of the new physical constants. But he might have seen in it a hint of something good to come. He would have liked, I think, that the theory is completely determined by the equations, which do not have to be talked away from time to time. He would have liked the complete absence of particles from the theory, and yet the emergence of 'particle tracks', and more generally of the 'particularity' of the world, on the macroscopic level. He might not have liked the GRW jumps, but he would have disliked them less than the old quantum jumps of his time. And he would not have been at all disturbed by their indeterminism. For as early as 1922, following his teacher Exner, he was expecting the fundamental laws to be statistical in character: '...once we have discarded our rooted predilection for absolute Causality, we shall succeed in overcoming the difficulties...' (Schrödinger, 1957).

For myself, I see the GRW model as a very nice illustration of how quantum mechanics, to become rational, requires only a change which is very small (on some measures!). And I am particularly struck by the fact that the model is as Lorentz invariant as it could be in the nonrelativistic version. It takes away the ground of my fear that any exact formulation of quantum mechanics must conflict with fundamental Lorentz invariance.

Appendix

Let

$$P(f;\mathbf{x}_m,n_m,t_m;\ldots\mathbf{x}_1,n_1,t_1;i)\mathrm{d}^3\mathbf{x}_1\ldots\mathrm{d}^3\mathbf{x}_m\,\mathrm{d}t_1\ldots\mathrm{d}t_m \tag{A.1}$$

be the probability that between some time i and some later time f there are m jumps, with the first at time t_1 in the interval $\mathrm{d}t_1$, involving argument $\mathbf{r}_{n_1}$, and centred at $\mathbf{x}_1$ in $\mathrm{d}^3\mathbf{x}_1$; and with the second at time t_2, involving argument $\mathbf{r}_{n_2}$, centred at $\mathbf{x}_2$, ... and so on. Then, from the basic assumptions,

$$P=\exp\lambda N(i-f)\langle i|E^+(f,i)E(f,i)|i\rangle, \tag{A.2}$$

where N is the total 'particle number', $|i\rangle$ denotes the initial state

$$|i\rangle=\psi(i,\mathbf{r}_1,\mathbf{r}_2\ldots) \tag{A.3}$$

and

$$E(f,i)=U(f,t_m)j(n_m,\mathbf{x}_m)\ldots U(t_2,t_1)j(n_1,\mathbf{x}_1)U(t_1,i) \tag{A.4}$$

with

$$U(s,t)=T\exp\int_s^t \mathrm{d}t'\,\frac{H(t')}{i\hbar} \tag{A.5}$$

and

$$j(n,x)=\lambda^{1/2}j(\mathbf{x}-\mathbf{r}_n). \tag{A.6}$$

In (A.5) we allow that the Hamiltonian might be time dependent, and so have a time-ordered product. Note the unitarity relation

$$U^+U = 1. \tag{A.7}$$

The leftmost U in (A.4) is actually redundant in (A.2), because of (A.7), but it is convenient later. The exponential in front of (A.2) arises from a product of exponentials

$$\exp - \lambda N(t' - t),$$

which are the probabilities of having no jumps in the corresponding time intervals. The formulae could be simplified somewhat by introducing Heisenberg operators, but we will not do so here.

Let us calculate from (A.1)–(A.4), for given i, the conditional probability distribution for jumps in the interval i' till f when the jumps between i and i' are given. We have only to divide (A.1) by the probability for the given jumps:

$$\exp \lambda N(i - i')|R|^2 \mathrm{d}^3\mathbf{x}_1 \ldots \mathrm{d}t_1 \ldots \tag{A.8}$$

with, from (A.2),

$$|R|^2 = \langle i|E^+(i',i)E(i',i)|i\rangle. \tag{A.9}$$

The result may be expressed in terms of

$$|i'\rangle = \frac{E(i',i)|i\rangle}{R} \tag{A.10}$$

when we note the factorization property

$$E(f,i) = E(f,i')E(i',i). \tag{A.11}$$

If we renumber the jumps in the reduced interval after i' to begin again with 1, we find again just (A.1)–(A.4) with i replaced everywhere by i'. So this was only a rather elaborate consistency check. But the manipulations involved will be useful for another purpose in a moment.

Let us now calculate from (A.1)–(A.4), with fixed f, the probability P' for jumps specified only up to some earlier time f', regardless of what happens later. To do so we have to sum over all possibilities in the interval between f' and f. There might be 0, 1, 2,... extra jumps in that remaining interval. The probability of the given jumps in the reduced interval, and no jumps in the remainder, is given directly by (A.2), which we rewrite as

$$X_0 \exp \lambda N(i - f')\langle i|E^+(f',i)E(f',i)|i\rangle \tag{A.12}$$

with
$$X_0 = \exp \lambda N(f' - f). \tag{A.13}$$

With one extra jump, E^+E in the expectation value is replaced by
$$E^+U^+|j(n,x)|^2 UE, \tag{A.14}$$

where the extra factor U evolves the system from time f' till the time t of the extra jump (n, x). Integration over x, using (4), replaces $|j(n,x)|^2$ by λ. The extra U^+U then goes away by unitarity. Summation over n gives a factor N, and integration over time t gives a factor $(f - f')$. Then the total one extra jump contribution to P' is (A.12) with X_0 replaced by
$$X_1 = \lambda N(f - f') \exp \lambda N(f' - f). \tag{A.15}$$

Proceeding in this way we find for the n-extra-jump contribution to P' again (A.11) but with X_0 replaced by
$$X_n = \frac{(\lambda N(f - f'))^n}{n!} \exp \lambda N(f' - f). \tag{A.16}$$

The factor $n!$ arises from the restriction of the multiple time integral to chronological order. To obtain the total P' we have to sum these n-extra-jump contributions over all n. This is easy, for
$$\sum X_n = 1. \tag{A.17}$$

The result for P' is just (A.1)–(A.4) with f replaced by f'. This is only as expected, but similar manipulations will be useful below.

Suppose now that the system falls into two noninteracting subsystems, with commuting Hamiltonians A and B, respectively:
$$H = A + B. \tag{A.18}$$

Then the operators U factorize:
$$U(t', t) = V(t', t)W(t', t) \tag{A.19}$$

with V and W constructed like U in (A.5), but with A and B replacing H. Since V and W commute, we can collect together the factors referring to each subsystem in (A.2), with the result
$$P = \exp \lambda L(i - f) \exp \lambda M(i - f) \langle i|F^+FG^+G|i\rangle, \tag{A.20}$$

where F and G are constructed like E in (A.4) but with operators of the first and second subsystems, respectively. The integers L and M are the 'particle numbers' of the subsystems:
$$L + M = N. \tag{A.21}$$

At this stage the initial and final times i and f are common to the two subsystems. But by the manipulations described above we can pass from i and f to later initial times, and earlier final times. Moreover, because the jump and evolution operators commute with one another, and have been collected together into separate commuting factors F and G, this can be done independently for the two subsystems. So we can take independent initial times i' and i'', and independent final times f' and f'', for the two subsystems, respectively.

The resulting probability distribution, over jumps in the reduced time intervals, is

$$P(f',f'';\mathbf{x}_m,n_m,t_m,\ldots\mathbf{x}_1,n_1,t_1;i',i'')\mathrm{d}^3\mathbf{x}_1\ldots\mathrm{d}^3\mathbf{x}_m\mathrm{d}t_1\ldots\mathrm{d}t_m. \quad (A.22)$$

where

$$P=\exp\lambda L(i'-f')\exp\lambda M(i''-f'')\langle i',i''|F^+FG^+G|i',i''\rangle. \quad (A.23)$$

The jumps and evolutions before i' and i'', in the two subsystems, respectively, have been incorporated into the initial state $|i',i''\rangle$. The jumps and evolutions in the reduced intervals, i' till f' and i'' till f'', make F and G, as in (A.4).

Note finally that if we are interested only in what happens in subsystem 1, we can sum over all possibilities for the second system in a now familiar way. The result is just (A.22), with reference to jumps in system 1 only, and (A.23) without any operator G. It is equivalent to

$$P=\mathrm{trace}_1 F^+F\rho, \quad (A.24)$$

where the trace is over the state space of system 1, and

$$\rho=\mathrm{trace}_2|i',i''\rangle\langle i',i''| \quad (A.25)$$

with the trace over the state space of system 2.

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