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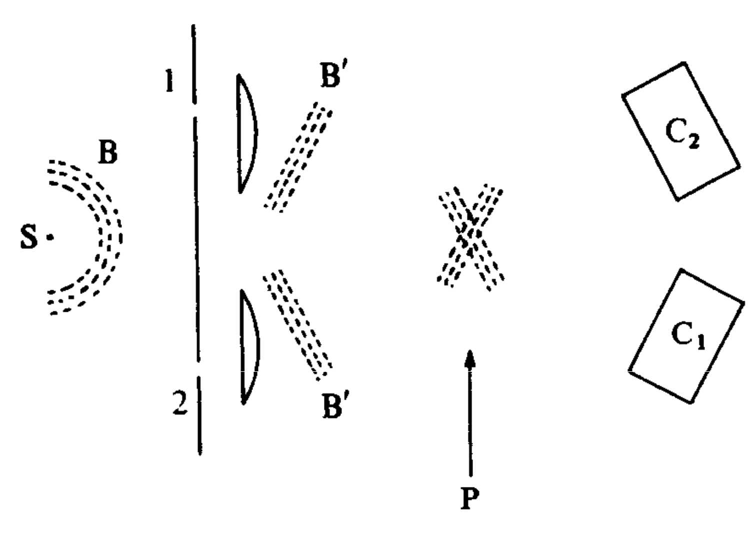
de Broglie–Bohm, delayed-choice double-slit experiment, and density matrix

I will try to interest you in the de Broglie[1]–Bohm[2] version of non-relativistic quantum mechanics. It is, in my opinion, very instructive. It is experimentally equivalent to the usual version insofar as the latter is unambiguous. But it does not require, in its very formulation, a vague division of the world into 'system' and 'apparatus,' nor of history into 'measurement' and 'nonmeasurement.' So it applies to the world at large, and not just to idealized laboratory procedures. Indeed the de Broglie–Bohm theory is sharp where the usual one is fuzzy, and general where the usual one is special. This is not a systematic exposition[3], but only an illustration of the ideas with a particularly nice example, and then some remarks on the role of the density matrix – in tribute to the title of this conference.

No one more eloquently than John A. Wheeler[4] has presented the delayed-choice double slit experiment. A pulsed particle source S (see Fig. 1) is so feeble that not more than one particle is emitted per pulse. The associated wave pulse B falls on a screen with slits 1 and 2. The

Fig. 1. A de Broglie wave pulse B from a particle source S traverses a screen with slits 1 and 2. The waves B′ emerging from the slits are focussed by lenses on particle counters C_1 and C_2. A photographic plate P may or may not be pushed into the interference region.



transmitted pulses B′ are focussed by off-centred lenses into intersecting plane wave trains which fall finally on particle counters C_1 and C_2 – unless a photographic plate P is pushed into the region where the two wave trains interpenetrate. The decision, to interpose the plate or not, is made only *after* the pulse has passed the slits. As a result of this choice the particle *either* falls on one of the two counters, indicating passage through one of the two slits, *or* contributes one of the spots on the photographic plate building an interference pattern after many repetitions. Sometimes the interference pattern is held to imply 'passage of the particle through both slits' – in some sense. Here it seems possible to *choose, later*, whether the particle, *earlier*, passed through one slit or two! Perhaps it is better not to think about it. 'No phenomenon is a phenomenon until it is an observed phenomenon.'

Consider now the de Broglie–Bohm version. To the question 'wave or particle?' they answer 'wave *and* particle.' The wave $\psi(t, \mathbf{r})$ is that of wave mechanics – but conceived, in the tradition of Maxwell and Einstein, as an objective field, and not just as some 'ghost wave' of information (of some presumably well-informed observer?). The particle rides along on the wave at some position $\mathbf{x}(t)$ with velocity.

$$\dot{\mathbf{x}}(t) = \frac{1}{m}\frac{\partial}{\partial \mathbf{r}} \operatorname{Im} \log \psi(t, \mathbf{r})|_{\mathbf{r}=\mathbf{x}} \tag{1}$$

This equation has the property that a probability distribution for $\mathbf{x}$ at time t

$$d^3\mathbf{x}\,|\psi(t, \mathbf{x})|^2$$

evolves into a distribution

$$d^3\mathbf{x}\,|\psi(t', \mathbf{x})|^2$$

at time t'. It is *assumed* that the particles are so delivered initially by the source, and then the familiar probability distribution of wave mechanics holds automatically at later times. Note that the only use of probability here is, as in classical statistical mechanics, to take account of uncertainty in initial conditions.

In this picture the wave always goes through both slits (as is the nature of waves) and the particle goes through only one (as is the nature of particles). But the particle is guided by the wave toward places where $|\psi|^2$ is large, and away from places where $|\psi|^2$ is small. And so if the plate is in position the particle contributes a spot to the interference pattern on the plate, or if the plate is absent the particle proceeds to one of the counters. In neither case is the earlier motion, of either particle or wave, affected by the later insertion or noninsertion of the plate. Clearly the particle pursues a bent path in the region where the wave trains interpenetrate[7]. It is vital here to put away the classical prejudice that a particle moves on a straight path in 'field-free' space – free, that is, from fields other than the de Broglie–Bohm! Indeed (in the absence of the plate) a particle passing through slit 1 falls finally not on counter C_1 but on C_2, and vice versa! It is clear just by symmetry that on the symmetry plane the perpendicular component of $\dot{\mathbf{x}}$ vanishes. The particle does not cross this plane. The naive classical picture has the particle, arriving on a given counter, going through the wrong slit.

Suppose next that detectors are added to the setup just behind slits 1 and 2 to register the passage of the particles. If we wish to follow the story after these detectors have or have not registered we cannot pretend that they are passive external devices (as we did for screen and lenses). They have to be included in the system. Consider then an initial wavefunction

$$\Psi(0) = \psi(0, \mathbf{r}) D_1^0(0, \mathbf{r}_1, \ldots) D_2^0(0, \mathbf{r}_2, \ldots)$$

where D_1^0 and D_2^0 are many-body wavefunctions for undischarged counters. Solution of the many-body Schrödinger equation yields a wavefunction

$$\Psi(t) = \Psi_1(t) + \Psi_2(t)$$

$$\left.\begin{aligned} \Psi_1(t) &= \psi_1(t, \mathbf{r}) D_1^1(t, \mathbf{r}_1, \ldots) D_2^0(t, \mathbf{r}_2, \ldots) \\ \Psi_2(t) &= \psi_2(t, \mathbf{r}) D_1^0(t, \mathbf{r}_1, \ldots) D_2^1(t, \mathbf{r}_2, \ldots) \end{aligned}\right\} \tag{2}$$

where the ψs are the two plane wave trains and the D^1s are wavefunctions for discharged counters. Let us suppose that a discharged counter pops up a flag saying 'yes' just to emphasize that it is a macroscopically different thing from an undischarged counter, in a very different region of configuration space.

The many-particle generalization of (1) gives for the particle of particular interest

$$\dot{\mathbf{x}}(t) = \frac{1}{m} \frac{\partial}{\partial \mathbf{r}} \operatorname{Im} \log \Psi(t, \mathbf{r}, \mathbf{r}_1, \mathbf{r}_2, \ldots) \Bigg|_{\substack{\mathbf{r} = \mathbf{x}(t) \\ \mathbf{r}_1 = \mathbf{x}_1(t), \text{etc.} \\ \mathbf{r}_2 = \mathbf{x}_2(t), \text{etc.}}} \tag{3}$$

Evaluation of this requires, in general, specification of not only $x(t)$ but also of the positions of all other particles. However, in the simple case of (2) the positions of other particles are sufficiently specified by 'detector 1 has discharged' or 'detector 2 has discharged.' The configurations so described are so different (grossly, macroscopically, so) that then only either Ψ_1 *or* Ψ_2 is significantly different from zero. Moreover, since for either Ψ_1 or Ψ_2 the variable **r** appears only in the factor ψ_1 or ψ_2, the complicated formula (3) reduces to the simple formula (1):

$$\dot{\mathbf{x}} = \frac{1}{m}\frac{\partial}{\partial \mathbf{x}} \operatorname{Im} \log \psi_1(t, \mathbf{x}) = \mathbf{v}_1$$

or

$$\dot{\mathbf{x}} = \frac{1}{m}\frac{\partial}{\partial \mathbf{x}} \operatorname{Im} \log \psi_2(t, \mathbf{x}) = \mathbf{v}_2$$

where $\mathbf{v}_1$ and $\mathbf{v}_2$ are velocities associated with the two plane wave trains.

This reduction from Ψ_1 *and* Ψ_2 to Ψ_1 *or* Ψ_2, on partial (macroscopic) specification of the configuration to be considered, illustrates the 'reduction of the wavefunction' in the de Broglie–Bohm picture. It is a purely theoretical operation and one need not ask just when it happens and how long it takes. The theorist does it when he finds it convenient.

The further reduction from Ψ_1 or Ψ_2 to ψ_1 or ψ_2 is a reduction from many particles to a few (one in this case). It illustrates how with a partial specification of the world at large it becomes possible in practice to deal with a small quantum system – although in principle the correct application of the theory is to the world as a whole. We made such a reduction of the system tacitly in the beginning when we said that certain screens and lenses, etc., were in position, but did not include them or the world at large in the quantum system. Note that in the de Broglie–Bohm scheme this singling out of a 'system' is a practical thing defined by circumstances, and is not already in the fundamental formulation of the theory.

Consider now the density matrix. When it is specified that counter 1 (say) has discharged, the conventional one-particle density matrix (with disregard of trivial normalization factors) is

$$\rho(\mathbf{x}, \mathbf{x}') = \psi_1(\mathbf{x})\psi_1^*(\mathbf{x}')$$

and the velocity $\dot{\mathbf{x}}_1 = \mathbf{v}_1$ is given equally by (1) or

$$\dot{\mathbf{x}}_1 = \frac{1}{m} \operatorname{Im} \left\{ [\rho(\mathbf{x}, \mathbf{x}')]^{-1} \frac{\partial}{\partial \mathbf{x}} \rho(\mathbf{x}, \mathbf{x}') \right\}_{\mathbf{x}=\mathbf{x}'} \tag{4}$$

But this is a rather trivial case. When it is not specified which counter has discharged the conventional density matrix is

$$\rho(\mathbf{x},\mathbf{x}') = \int d^3\mathbf{x}_1 d^3\mathbf{x}_2 \ldots \Psi(\mathbf{x},\mathbf{x}_1,\mathbf{x}_2,\ldots)\Psi^*(\mathbf{x}',\mathbf{x}_1,\mathbf{x}_2\ldots)$$

$$= \psi_1(\mathbf{x})\psi_1^*(\mathbf{x}') + \psi_2(\mathbf{x})\psi_2^*(\mathbf{x}')$$

I do not see how to recover from this the fact that we have (nearly always) velocity either $\mathbf{v}_1$ or $\mathbf{v}_2$. Naive application of (4) gives something else. So in the de Broglie–Bohm theory a fundamental significance is given to the wavefunction, and it cannot be transferred to the density matrix. This is here illustrated for the one-particle density matrix, but it equally so for the world density matrix if a probability distribution over world wavefunctions is considered. Of course the density matrix retains all its usual practical utility in connection with quantum statistics.

That the above treatment of the detectors was greatly oversimplified does not affect the main points made. Real detectors would respond in a variety of ways to particles traversing in a variety of ways. Not only would ψ_1 and ψ_2 become incoherent, but each would be replaced by many incoherent parts.

That the theory is supposed to apply fundamentally to the world as a whole requires ultimately that any 'observers' be included in the system. This raises no particular problem so long as they are conceived as not essentially different from computers, equipped perhaps with 'random' number generators. Then everything is in fact predetermined at the fundamental level – including the 'late' decision whether to insert the plate. To include creatures with genuine free will would require some development, and here the de Broglie–Bohm version might develop differently from the usual approach. To make the issue experimental would require identification of situations in which the differences betweeen computers and free agents were essential.

That the guiding wave, in the general case, propagates not in ordinary three-space but in a multidimensional-configuration space is the origin of the notorious 'nonlocality' of quantum mechanics[5]. It is a merit of the de Broglie–Bohm version to bring this out so explicitly that it cannot be ignored.[6]

References

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