

Groups and Representations

Homework Assignment 14 (due on 7 February 2024)

Problem 46

Consider Young diagrams with row lengths $\lambda = (\lambda_1, \dots, \lambda_N)$, and $\lambda' = (\lambda_1 + k, \dots, \lambda_N + k)$, $k \geq 1$. Show that the $SU(N)$ -irreps Γ^λ and $\Gamma^{\lambda'}$ are equivalent.

HINT: Use the Littlewood-Richardson rule and the result of Problem 45.

Problem 47

Let Γ^λ be an $SU(3)$ -irrep with Young diagram λ . Determine how often Γ^λ appears in the product rep $\lambda \otimes \begin{array}{|c|} \hline \square \\ \hline \end{array}$.

HINT: Study separately the cases of rectangular Young diagrams λ (with one or two rows) and of non-rectangular diagrams.

Problem 48

Decompose the product rep $\square \otimes \square \otimes \square$ of $SU(3)$ into irreps. Use the notation of Problem 35 (e.g. $|uds\rangle = |u\rangle \otimes |d\rangle \otimes |s\rangle \in \square^{\otimes 3}$) and explicitly construct bases for the irreducible invariant subspaces. Compare with the results of Problem 35. What is the relation between the irreducible subspaces with respect to $SU(3)$ and those with respect to S_3 ?