Groups and Representations

Homework Assignment 11 (due on 17 January 2024)

Problem 40

a) Determine the Haar measure for SU(2) in axis-angle parametrisation,

$$U = \exp\left(-\mathrm{i}\tfrac{\alpha}{2}\vec{\sigma}\cdot\vec{x}\right),$$

with $0 \leq \alpha \leq 2\pi$ and $\vec{x} \in S^2 \hookrightarrow \mathbb{R}^3$. Normalise s.t. vol(SU(2)) = 1.

HINT: It is convenient to first show $(\vec{x}\cdot\vec{\sigma})(\vec{y}\cdot\vec{\sigma}) = \mathbb{1}\vec{x}\cdot\vec{y} + \mathrm{i}\vec{\sigma}(\vec{x}\times\vec{y})$ and to use the unit vectors $\vec{e}_r, \vec{e}_\vartheta, \vec{e}_\varphi$ for spherical coordinates.

b) Use the result of (a) together with the results of Problem 37 in order to determine the Haar measure for SO(3) in the axis-angle parametrisation.