

# Groups and Representations

Homework Assignment 1 (due on 25 October 2023)

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**Problem 1** (rearrangement lemma)

For the group table of a finite group prove that no two elements within one row (or column) can be the same.

**Problem 2**

List all possible groups (up to isomorphism) of order 3 and 4 by explicitly constructing their group tables.

**Problem 3**

Show that the map  $\exp : (\mathbb{R}, +) \rightarrow (\mathbb{C} \setminus \{0\}, \cdot)$ ,  $t \mapsto e^{2\pi it}$  is a group homomorphism. Determine kernel and image of  $\exp$ , and check whether these are subgroups of  $(\mathbb{R}, +)$  and  $(\mathbb{C} \setminus \{0\}, \cdot)$ , respectively.

**Problem 4**

Let

$$\mathcal{B} = \{f : \mathbb{R} \rightarrow \mathbb{R} \mid f \text{ bijective}\} \quad \text{and}$$
$$\mathcal{A} = \{f : \mathbb{R} \rightarrow \mathbb{R} \mid \exists a, b \in \mathbb{R}, a \neq 0 : f(x) = ax + b\} .$$

Show that  $(\mathcal{B}, \circ)$ , with  $\circ$  the composition of functions, is a group and that  $\mathcal{A}$  is a subgroup of  $\mathcal{B}$ . Is  $\mathcal{B}$  or  $\mathcal{A}$  abelian?

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