UNIVERSITÄT TÜBINGEN
FACHBEREICH MATHEMATIK

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Convex Geometry

Winter term 2024/25

Exercise sheet 1

Due on: Thursday, 31.10.2024, 10:00

Exercise 1 (5 Points)

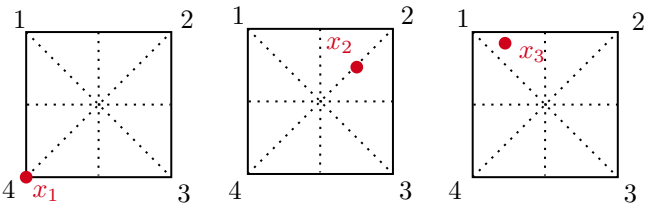
Consider the group

$$D_4 := \{id,\ (1234), (13)(24), (1432), (12)(34), (14)(23), (24), (13)\} \subset \mathbb{S}_4.$$

Let $M := \{1, 2, 3, 4\}$ be the vertex set of a square in $\square \subset \mathbb{R}^2$. Then D_4 acts on M via $\sigma \cdot i := \sigma(i)$ for $\sigma \in D_4$ and $i \in M$. This action extends to an action on all points of the square.

(a) Determine $Stab(x_i)$ for $i = 1, 2, 3$ (see Figure).

(b) Does there exist an $x \in \square$ such that $|Stab(x)| = 4$?



Exercise 2 (6 Points)

Consider the affine subspaces $W_1, W_2 \subset \mathbb{A}(\mathbb{R}^5)$ defined by

$$W_1 := \begin{pmatrix} 1 \\ 0 \\ 2 \\ 9 \\ 0 \end{pmatrix} + \left\langle \begin{pmatrix} 1 \\ 0 \\ 2 \\ 1 \\ -2 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 0 \\ 1 \\ 0 \end{pmatrix} \right\rangle \text{ and } W_2 := \begin{pmatrix} 0 \\ 0 \\ 1 \\ -1 \\ -1 \end{pmatrix} + \left\langle \begin{pmatrix} 0 \\ -1 \\ -1 \\ 0 \\ -1 \end{pmatrix}, \begin{pmatrix} 0 \\ -1 \\ 0 \\ -1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ -4 \\ -2 \\ -1 \end{pmatrix} \right\rangle$$

and determine $W_1 \cap W_2$.

Exercise 3 (6 Points)

Let $W_1, W_2 \subset \mathbb{A}(\mathbb{R}^n)$ be two planes. What are the possible dimensions of the intersection $W_1 \cap W_2$ when $n = 3$ and $n = 4$? Give an example for each possibility.

Exercise 4 (2 Points)

Explain why a chair (in $\mathbb{R}^3$) with 3 legs never wobbles, but a chair with 4 legs might.

Hand in via URM. Exercise classes take place on Wednesdays 12-14, in S11.