

Exercise sheet 6

Due on: Thursday, 5.12.2024, 10:00

Exercise 1 (4 Points)

Let τ be a face of a cone σ . Show: If $v, w \in \sigma$ and $v + w \in \tau$ holds, then also $v, w \in \tau$.

Exercise 2 (6 Points)

The relative interior $\text{Relint}(\sigma)$ of a cone σ is the topological interior within the linear space $\text{span}(\sigma)$.

Let u_1, \dots, u_n be generators of σ and suppose that for all i the ray $\{\lambda u_i | \lambda \geq 0\}$ is an edge of σ . Show:

$$\begin{aligned} u \in \text{Relint}(\sigma) &\Leftrightarrow \\ u = \sum \lambda_i u_i \text{ for some } \lambda_i > 0 \ \forall i &\Leftrightarrow \\ \langle m, u \rangle > 0 \ \forall m \in \sigma^\vee \setminus \sigma^\perp, & \end{aligned}$$

where σ^\perp denotes the annihilator of σ , i.e. the set of all m that satisfy $\langle m, u \rangle = 0$ for all $u \in \sigma$.

Exercise 3 (4 Points)

Let σ be a strictly convex, full-dimensional cone. The following example shows: σ and σ^\vee do not necessarily have the same number of edges. Let $\sigma \subset \mathbb{R}^4$ be the cone generated by $2e_i + e_j$, $1 \leq i, j \leq 4$, $i \neq j$.

- Show: σ has 12 edges.
 - Show: σ^\vee is generated by e_i and $-e_i + 2\sum_{j \neq i} e_j$, $1 \leq i \leq 4$ and has 8 edges.
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Exercise 4 (4 Points)

Determine the face lattice of the 3-cube and the 4-cube. *Hint:* You can use Exercise 3 from Sheet 5.

Hand in via URM. Exercise classes take place on Wednesdays 12-14, in S11.