Convex Geometry
Winter term 2024/25

Exercise sheet 5   Due on: Thursday, 28.11.2024, 10:00

Exercise 1   (4+2=6 Points)

Let $P \subset \mathbb{R}^d$ and $Q \subset \mathbb{R}^e$ be two polytopes. The *product* $P \times Q$ of P and Q is a polytope of dimension $\dim(P) + \dim(Q)$ defined by $P \times Q := \{(p,q) \mid p \in P, q \in Q\} \subset \mathbb{R}^{d+e}$.

(a) Show: Faces of $P \times Q$ are of the form $P' \times Q'$, where P' is a face of P and Q' is a face of Q.

(b) Let $P = \operatorname{conv}((0,0),(1,0),(0,1)\} \subset \mathbb{R}^2$ and $Q = \operatorname{conv}(0,1) \subset \mathbb{R}$. Determine the face lattices of $P \times Q$ and of $P \times P$ (i.e. the set of all faces partially ordered by inclusion, see also Exercise 1 on Sheet 4).

Exercise 2   (6 Points)

For $i = 1, 2$ consider the polyhedron $P_i := \{x \in \mathbb{R}^2 \mid A_i x \leqslant b_i\}$, where

$$(A_1|b_1) = \begin{pmatrix} -2 & 1 & 2 \\ 1 & -2 & 2 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \\ -1 & -1 & -1 \\ -1 & -1 & 0 \\ -3 & 1 & 2 \end{pmatrix} \text{ and } (A_2|b_2) = \begin{pmatrix} -2 & 1 & 2 \\ 2 & -1 & 4 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \\ -1 & -1 & -1 \\ -1 & -1 & 0 \\ -3 & 1 & 2 \end{pmatrix}.$$

Write P_1 and P_2 as $\operatorname{conv}(V) + \operatorname{cone}(Y)$ for $V, Y \subset \mathbb{R}^2$ as small as possible.

Exercise 3   (6 Points)

Find the H-description of the d-cube $\operatorname{conv}(\{1, -1\}^d)$. Show: The faces are in bijection with vectors in $\{+, -, 0\}^d$.

Hand in via URM. Exercise classes take place on Wednesdays 12-14, in S11.