UNIVERSITÄT TÜBINGEN
FACHBEREICH MATHEMATIK

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Convex Geometry

Winter term 2024/25

Exercise sheet 4

Due on: Thursday, 21.11.2024, 10:00

Exercise 1 (4 Points)

Let $v_1, ..., v_4$ be affinely independent points in $\mathbb{R}^3$. A *tetrahedron* T is a polytope given by the convex hull of its 4 vertices, i.e. $T = conv(v_1, ..., v_4) \subset \mathbb{R}^3$ (see Figure). Let M be the set containing the 4 vertices, the 6 edges and the 4 faces of T. Then M is a partially ordered set via

$$m_1 \leqslant m_2 :\Leftrightarrow m_1 \subset m_2, \text{ where } m_1, m_2 \in M.$$

Draw the Hasse diagram of $(M, \leqslant)$.
Hint: A Hasse diagram is a graph with vertex set M. Two vertices m_1 and m_2 are connected by an edge if $m_1 \leqslant m_2$ and there is no m_3 distinct from m_1 and m_2 such that $m_1 \leqslant m_3 \leqslant m_2$.

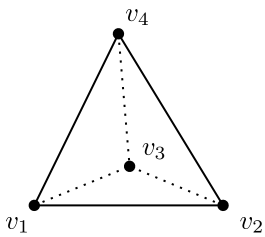

Figure 1: A tetrahedron in $\mathbb{R}^3$.

Exercise 2 (6 Points)

Let $P \subset \mathbb{R}^3$ be a 3 dimensional polytope such that every pair of vertices is connected by an edge. Show that P is a Tetrahedron in the sense of exercise 1.

Exercise 3 (4 Points)

Follow the Polymake tutorial for polytopes: `https://polymake.org/doku.php/user_guide/tutorials/apps_polytope`. Define a 3 dimensional unit cube c as the convex hull of finitely many points (V-description) and use Polymake to convert the V-description of c to its H-description. Take a screenshot of your results.
Hint: You can use the online version of Polymake: `https://polymake.org/doku.php/start`. To start a session, don't forget to enter 'polymake'. Note that not all methods (e.g. the 'VISUAL' method) are available when using the online version.

Exercise 4 (6 Points)

Let $\sigma := cone(\begin{pmatrix}1\\0\\1\end{pmatrix}, \begin{pmatrix}0\\1\\1\end{pmatrix}, \begin{pmatrix}1\\1\\1\end{pmatrix}) \subset \mathbb{R}^3$. Determine the dual cone $\sigma^\vee$.

Hand in via URM. Exercise classes take place on Wednesdays 12-14, in S11.