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FACHBEREICH MATHEMATIK Lou-Jean Cobigo

Convex Geometry

Winter term 2024/25

Exercise sheet 2

Due on: Thursday, 07.11.2024, 10:00

Exercise 1 (4 Points)

Let $f : \mathbb{A}(\mathbb{R}^2) \to \mathbb{A}(\mathbb{R}^2)$ be an affine map. The set $F := \{x \in \mathbb{R}^2 \mid f(x) = x\}$ is called the *fixed point set* of f. Show that the following are the only possible cases:

1. $F = \emptyset$, i.e. f does not have a fixed point.
2. $|F| = 1$, i.e. f has exactly one fixed point.
3. $|F| = \infty$ and F is a line.
4. $F = \mathbb{R}^2$.

Exercise 2 (6 Points)

Let $\mathbb{A}(V)$ be the affine space of the vector space V, $H_1, H_2, H_3 \subset \mathbb{A}(V)$ any three distinct parallel hyperplanes and $L_1, L_2 \subset \mathbb{A}(V)$, a pair of lines not parallel to H_i. Denote by a_i the intersection point of L_1 with H_i and by b_i the intersection point of L_2 with H_i and show that:

$$\frac{a_1a_3}{a_1a_2} = \frac{b_1b_3}{b_1b_2},$$

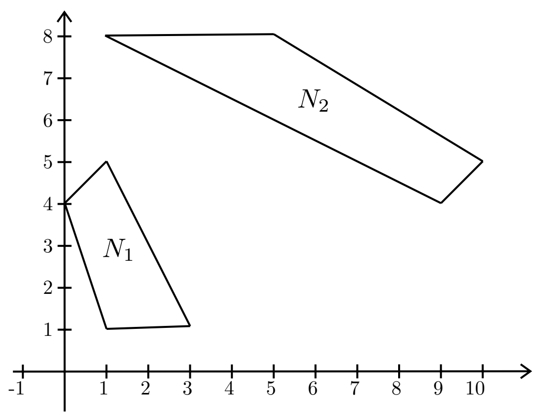
where $\frac{a_1a_3}{a_1a_2}$ $(\frac{b_1b_3}{b_1b_2})$ denotes the ratio of a_2 and a_3 with respect to a_1 (the ratio of b_2 and b_3 with respect to b_1).
Hint: Consider the affine map $f : \mathbb{A}(V) \to \mathbb{A}(V)$ whose linear part sends the subspace of translations H that is common to all H_i to 0 and that maps b_1 to a_1.

Exercise 3 (2 +2 = 4 Points)

Consider the subsets

- $E_1, E_2 \subset \mathbb{R}^2$ given by $E_1 := \{(x, y) \in \mathbb{R}^2 \mid 4x^2 + y^2 - 4(2x + y) + 4 = 0\}$ and
  $E_2 := \{(x, y) \in \mathbb{R}^2 \mid 4x^2 + y^2 - 4 = 0\}$,
- $N_1, N_2 \subset \mathbb{R}^2$ shown in the Figure,

and find affine maps $f, g : \mathbb{A}(\mathbb{R}^2) \to \mathbb{A}(\mathbb{R}^2)$ with $f(E_1) = E_2$ and $g(N_1) = N_2$.



Exercise 4 (4 Points)

Let V be a vector space of dimension n and let $a_0, ..., a_n \in \mathbb{A}(V)$ be affinely independent. Show: An affine map $f : \mathbb{A}(V) \to \mathbb{A}(V)$ is uniquely determined by the images $f(a_0), ..., f(a_n)$.

Hand in via URM. Exercise classes take place on Wednesdays 12-14, in S11.